Green's function of quasiparticles in Calogero model and quantum hydrodynamics

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Using a hydrodynamic description, we study the properties of the one-dimensional Calogero-Sutherland model. The correlation function of quasiparticle excitations (anions) is calculated by the instanton technique. The instanton trajectory is given by a solitary wave propagating through a quantum fluid. The smooth part of the quasiparticle correlation function far from the light cone oscillates as a function of coordinate.

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The goal of this work is to establish a connection between quantum hydrodynamics of one-dimensional interacting electrons and their correlation functions. Most of those properties are known from a simplified version of one-dimensional hydrodynamics, known as the Tomonaga-Luttinger (TL) model. Despite enormous success in treating low-dimensional physics, as was recently pointed out in Ref. 1, the TL model misses a number of important physical phenomena. These include photovoltaic effect, thermopower low temperature Coulomb drag, and other effects for which the breaking of particle-hole symmetry is necessary. Since conventional bosonization relies on the linearization of the single-particle spectrum, such phenomena can hardly be approached. That is why the extension of treatment from a simplified TL version of quantum hydrodynamics to its full nonlinear form seems necessary. While the hydrodynamics of electrons interacting via Coulomb potential still remains an open problem, some features of the hydrodynamics that arise from singular $1/r^2$ (Calogero-Sutherland[2]) interaction are known. Hydrodynamics of Calogero-Sutherland (CS) fluid had been studied in Refs. 3 and 4, but no attempt to bridge it with any of the correlation functions has ever been made. In this work, we show how some features of quantum hydrodynamical excitations correspond to observable correlation functions, in particular, to the quasiparticle Green's function and polarization operator.

The CS model is described by the Hamiltonian

$$H = -\frac{1}{2}\sum_{i=1}^{N} \partial_i^2 + \left(\frac{\pi}{L}\right)^2 \sum_{i>j}^{N} \frac{\lambda(\lambda-1)}{\sin^2[(\pi/L)(x_i - x_j)]}, \qquad (1)$$

and we use a convention ($\hbar=1$, $m=1$). A most striking property of this model is a special $(1/r^2)$ form of interaction. Despite its rather unusual form, the CS model has some flavor of universality on it because it appears at various "unrelated" areas, including spin chains, fractional quantum Hall effects, and disordered metals.[5,6] The interaction constant λ describes the strength of electron interaction. The values $\lambda=1$, 2, and 4 correspond to Dyson's symmetry classes of random matrix theory.[7]

The exact eigenstates and eigenfunctions of the CS model have been found, first, by Forrester[8] for integer values and, later on, by Ha[9] for rational values of coupling strength λ. The continuous description of the correlation function for generic values of λ was until recently absent. A certain progress had been reached in Ref. 10 based on the replica symmetry breaking technique. An asymptotic analysis of exact formulas for the density structure factor near the edges of its support was performed in Ref. 11. The connection between the position of the support of the structure function and the spectrum of quasiparticles and quasiholes was emphasized.

Here, we take a different route and focus on the hydrodynamics of CS fluid without resorting to its exact solution. Since a hydrodynamical description can be formulated for various forms of particle interactions, this method improves our understanding of many-body interacting electrons. It improves our chances to solve problems with generic interaction (such as Coulomb interaction), for which no exact results are known.

The structure of this paper is as follows. We bosonize the problem by using the collective variable approach.[3,12] We explain what part of correlation functions we are after on the example of free fermions. We calculate the Green's function (for the large value of the interaction strength) by using saddle-point approximation. Finally, we determine the support of the polarization operator based on the hydrodynamical calculations.

We consider a ring geometry of the system. The coordinates of the particles can be described by a complex variable $z_n = Le^{i\theta_n}$, where θ is an angle along the circle of circumference L. The Hamiltonian can be rewritten as

$$H = (2\pi)^2 \left[\frac{1}{2L^2}\sum_{j=1}^{N} (z_j\partial_j)^2 + \sum_{i\neq j}^{N} \frac{\lambda(\lambda-1)}{|z_i - z_j|^2}\right]. \qquad (2)$$

The ground state of the Hamiltonian (1) is

$$\Psi_0 = \left(\prod_{i=1}^{N} z_i\right)^{-\lambda(N-1)/2} |\Delta|^{\lambda-1}\Delta, \quad \Delta = \prod_{i<j}^{N} (z_i - z_j). \qquad (3)$$

The excited states

$$\Psi_\kappa = \Psi_0 \Phi_\kappa^B, \qquad (4)$$

where Φ_κ^B is a symmetric function of the particle coordinates parametrized by partition κ. The Hamiltonian $H = \Psi_0^{-1} H \Psi_0$ is acting on the space of the symmetric (bosonic) function. After some algebra, one arrives at

$$H = \sum_{i=1}^{N} D_i^2 + \lambda \sum_{i<j}^{N} \frac{z_i + z_j}{z_i - z_j}(D_i - D_j), \qquad (5)$$

where $D_i = z_i\partial_i$. To prepare for the second quantization, one defines so-called collective variables $p(\theta) = \Sigma_{i=1}^{N} \delta(\theta - \theta_i)$, and

$p_k = \int_0^{2\pi} d\theta e^{ik\theta} p(\theta)$. In terms of collective variables, the Hamiltonian has the form

$$H = \frac{1}{2}\sum_{m,n=-N}^{N} mnp_{n+m}\frac{\partial^2}{\partial p_n \partial p_m} + (1-\lambda)\sum_{n=-N}^{N} n^2 p_n \frac{\partial}{\partial p_n} + \frac{\lambda}{2}\sum_{m=0}^{N-1}\sum_{n=1}^{N-m}(n+m)\left[p_n p_m \frac{\partial}{\partial p_{n+m}} + p_{-n}p_{-m}\frac{\partial}{\partial p_{-n-m}}\right]. \tag{6}$$

So far, our calculation has been exact. Now, we move on to the hydrodynamic limit. In the limit of $N \to \infty$, the Hamiltonian Eq. (6) can be written as

$$H = \int dx \left[\frac{1}{2}\rho v^2 + U[\rho]\right]. \tag{7}$$

Here, x is a coordinate along the circle $\left(x = \frac{L}{2\pi}\theta\right)$, the linear density $\rho(x) = \frac{2\pi}{L}p(\theta)$, etc. The potential energy of quantum liquid is given by

$$U = \frac{\pi^2\lambda^2}{6}\rho^3 - \frac{\pi\lambda(\lambda-1)}{2}\rho^H \rho_x + \frac{(\lambda-1)^2}{8}\frac{(\rho_x)^2}{\rho}, \tag{8}$$

the Hilbert transform is defined as

$$\rho^H(x) = \frac{1}{\pi}P\int dx' \frac{\rho(x')}{x'-x}, \tag{9}$$

and a velocity operator is given by

$$v(x) = -i\left(\partial_x \frac{\delta}{\delta\rho(x)}\right) - i\pi\lambda\rho^H(x) - \frac{i}{2}(\lambda-1)\frac{\partial_x\rho(x)}{\rho(x)}. \tag{10}$$

It is easy to see that, with these definitions, particle velocity and density satisfy a standard commutation relation one would expect for any hydrodynamic theory,[13] $[v(x), \rho(y)] = -i\delta'(x-y)$. To reproduce the conventional bosonization theory, one expands the Hamiltonian up to second order in fluctuations (due to conservation laws, the velocity v and density operators $\delta\rho$ are of the same order) and spatial gradient (keeping only the terms that have no gradient). By doing so, we see that on the conventional Luttinger model, interaction enters only through the sound velocity of bosonic modes.

Now, we use the Hamiltonian (7) to study the effects of the spectrum curvature. We start with the single-particle Green's function

$$G(x,t) = -i\langle T\Psi(x,t)\Psi^\dagger(0,0)\rangle, \tag{11}$$

where Ψ is an original fermionic (anionic for $\lambda \neq 1$) field. As we will see shortly, the Green's functions for the CS model and for free fermions turn out somewhat similar. We consider the noninteracting case first. By the Wick theorem, the Green's function can be easily evaluated. For parabolic dispersion, the time ordered Green's function ($t > 0$) is given by

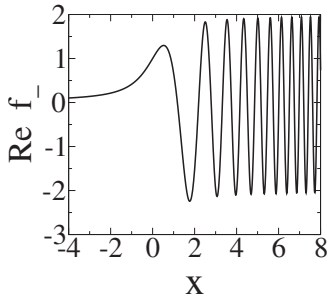

FIG. 1. $\mathrm{Re}\, f_-\left(\frac{x-v_F t}{\sqrt{2t}}\right)$ as a function of x for $v_F = 1$ and $t = 1$. The left tail ($x \ll -1$) is captured by a linear bosonization.

$$G(x,t) = -i\int_0^\infty dp[e^{ip_F x}e^{i(x-v_F t)p-(ip^2/2)t} + e^{-ip_F x}e^{-i(x+v_F t)p-(ip^2/2)t}], \tag{12}$$

where p_F is the Fermi momentum and v_F is the Fermi velocity. After performing an integration, one gets

$$G(x,t) = -\sqrt{\frac{\pi}{2t}}e^{i\pi/4}\left[e^{-ip_F x}f_+\left(\frac{x+v_F t}{\sqrt{2t}}\right) + e^{ip_F x}f_-\left(\frac{x-v_F t}{\sqrt{2t}}\right)\right], \tag{13}$$

where

$$f_\pm(x) \equiv e^{ix^2}\,\mathrm{Erfc}(\pm e^{i\pi/4}x). \tag{14}$$

The Green's function is a sum of left and right moving terms. Each one is a product of the smooth function ($f_\pm$) and fast oscillating term. The real part of the function f_- is plotted in Fig. 1. One notices that though the part on the left of the light cone is captured by linear bosonization [cf. Eq. (16)], the right part is not (the situation for f_+ is reversed). Thus, we see the linear bosonization limit is achieved only for parts of the x-t plane for a strictly parabolic spectrum.

Instead, the power law decay correlation function has a universal oscillating behavior. In the free electron case, the oscillating part of f may be found by performing a saddle-point approximation to an integral over momentum in Eq. (12),

$$G = -i\sqrt{\pi/t}[e^{ip_F x}e^{i(x-v_F t)^2/2t}\theta(x-v_F t) + e^{-ip_F x}e^{i(x+v_F t)^2/2t}\theta(-x-v_F t)]. \tag{15}$$

One sees that a saddle point gives a dominant contribution only if the particle moves faster than the Fermi velocity. In this region, the result is an oscillatory function. For the particles moving slower than the Fermi velocity, the saddle point lies out of the region of integration and the integral is determined by its low limit. In that case, we restore a standard power decay of the Green's function. For particles moving faster than the Fermi velocity, the particle does *not* agree with the linear bosonization prediction

$$G(x,t) = -(1/2\pi)[e^{-ip_F x}/(v_F t + x) + (R \leftrightarrow L)]. \tag{16}$$

The reason for the disagreement between the asymptotic analysis of the Green's function of free electrons and those

calculated in the bosonized theory is a regularization tacitly assumed in the latter. Surprisingly, the results of the TL theory are valid for electrons on the lattice, but not for those that have a strictly parabolic spectrum.

In real metals, electrons are subject to periodic potential. They form Bloch states and have a periodic spectrum that has a negative curvature close to the boundary of the Brillouin zone. This limits the maximally accessible velocity of the classical trajectory. The standard bosonization considers only the situation in which there is no classical trajectory available, thus opening a parametric window of long time, long range physics far from the light cone and yet unaccessible by means of the standard TL model. This can be most clearly demonstrated within a tight-binding model, in which a single-particle spectrum is given by $\epsilon(p)=W\left[1-\cos\left(\frac{pa}{2}\right)\right]$, where a is a lattice constant and W is a bandwidth. An integral over momentum [cf. Eq. (12)] has a stationary point at $\frac{2}{aW}\frac{x}{t}=\sin\left(\frac{pa}{2}\right)$. The right hand side of this equation is a periodic function of momentum and it is smaller than 1. So the saddle-point contribution disappears if the velocity on the classical trajectory exceeds the one available by the band structure $[aW \ll (x-v_F t)/t]$. This is a linear bosonization limit, intensively studied in the literature so far.[14,15] We will consider the situation when the classical trajectory dominates the result, but the distance to the light cone is still large $[v_F \ll (x-v_F t)/t \ll aW]$. In this case, the semiclassical approximation is justified, but linear bosonization is not. In the last remaining interval of points close to the light cone $[(x-v_F t)/t \ll v_F]$, the Wentzel-Kramers-Brillouin (WKB) approximation fails and different methods are needed. We stress that in all cases, the absolute values of times and coordinates, measured in the Fermi energy and wavelength, are large.

The rest of this Brief Report is devoted to the study of asymptotic properties of the Calogero model in the semiclassical regime.

One may note that a calculation of single-particle Green's function Eq. (11) involves summation over many-body states with one particle-hole excitation. Since exact many-body states for the CS model are known [Eq. (4)], it is natural to construct creation and annihilation operators that act in the Hilbert space of exact many-body states. Such operators in the first quantization formalism had been constructed in Ref. 16 for $\lambda=1/2$ and 2, and for rational values of the coupling constant.[17] Due to the analogy between the CS model and the fractional Hall effect, such excitations are referred to as anyons. One can show that an anyon creation operator in the hydrodynamical description is

$$\Psi_A(x,t) \simeq \exp\left(i\int_{-\infty}^{x} dx[v(x,t)+\pi\lambda\rho(x,t)]\right). \quad (17)$$

It is straightforward to verify that this definition is consistent[18] with the "fractional" commutation relations

$$\Psi_A(x)\Psi_A(x') = e^{-i\pi\lambda\ \mathrm{sgn}(x'-x)}\Psi_A(x')\Psi_A(x). \quad (18)$$

Despite its formal appearance, quasiparticles in the CS model are important. This excitation plays a crucial role in various particle-hole excitations in the CS model. In particular, it is shown below for a density-density correlation function. A semiclassical analysis of a decay of a single electron into a large number of quasiparticles was performed in Ref. 19. A calculation of the entire electron Green's function would be reported elsewhere.

The Green's function is given by a functional integral

$$G_A(x',t') = \frac{-i}{Z}\int \mathcal{D}\rho\mathcal{D}v e^{iS}\Psi_A(x',t')\Psi_A^{\dagger}(0,0). \quad (19)$$

The hydrodynamic action corresponding to the Hamiltonian Eq. (7) is given by

$$S = \int dxdt\left[-v\partial_x^{-1}\partial_t\rho - \frac{1}{2}\rho v^2 - U[\rho]\right]. \quad (20)$$

To calculate the Green's function, we use the steepest descent method, known also as WKB. The action (20) has massive direction and low energy (massless) excitations in the configuration space (such as constant shift $x\to x+a, t\to t+b$). The first may be integrated out by the saddle-point method. The latter gives rise to zero modes, which need to be integrated exactly. The saddle-point equations are

$$\rho_t + \partial_x(\rho v) = -\delta(x-x')\delta(t-t') + \delta(x)\delta(t), \quad (21)$$

$$v_t + vv_x + \partial_x(\delta U/\delta\rho) = -\pi\lambda\delta(x-x')\delta(t-t') + \pi\lambda\delta(x)\delta(t). \quad (22)$$

Due to the integrability of the original CS model, the resulting hydrodynamical set of equations is special. It contains an infinite number of integrals of motion and consequently exhibits soliton solutions. The connection between the dynamics of classical particles interacting via the $1/r^2$ potential and soliton solution of the Benjamin-Ono equation has been established in Ref. 20. It is known that for some integrable models, the remarkable correspondence between particles of the original fermionic model and solitons of the bosonized theory can be extended to the quantum limit.[21] For the quantum CS model, such a connection has been found in Ref. 4 and was recently stressed in Ref. 22. In this work, we use this correspondence to find the quasiparticle correlation function.

In the limit of large λ, we look for solitonic wave solutions $(0<t<t')$ for the density

$$\rho(x,t) = 1 + \rho_s(x - Vt - X(t)) \quad (23)$$

and velocity

$$v(x,t) = v_s(x - Vt - X(t)) \quad (24)$$

of the fluid. Here, $X(t)$ is a global variable. It corresponds to the motion of the liquid as a whole. This is a zero mode of the system. The soliton's density and velocity are given by[4]

$$\rho_s(x) = (B/\pi)[1/(x^2+B^2)],$$

$$v_s(x) = (Bv/\pi)[1/(x^2+B^2+B/\pi)]. \quad (25)$$

Here, the width of a soliton is

$$B = \pi\lambda^2/(V^2 - \pi^2\lambda^2) \tag{26}$$

and $V=x'/t'$ is its velocity. For a soliton to exist, its width has to be positive ($B>0$). In other words, the velocity has to *exceed* the sound velocity ($V \geqslant s$, $s=\pi\lambda\rho_0$). We notice a remarkable similarity with the saddle-point calculation of the ordinary integral Eqs. (13) and (15). In that case, the saddle-point contribution dominates the Green's function for supersonic trajectories, resulting in oscillatory x dependence.

In the case of a functional integral, the saddle-point solution is a field configuration localized in space. As the velocity of the soliton increases $v \gg s$, its width decreases ($B \to 0$) and its semiclassical interpretation as a particle becomes more and more justified.

By integrating over an infinitesimal region around $x=0$, $x=x'$ over x, we see that the solution (25), indeed, has a right singularity. The total number of electrons carried by the soliton

$$\int \rho(x)dx = 1 \tag{27}$$

is quantized to 1. The velocity of the soliton is not quantized. Then, from exponential accuracy, we get that the Green's function of the CS model is given by

$$G_A(x,t) \simeq e^{iS_{cl}} \int_{X(0)=0}^{X(t')=0} \mathcal{D}X \exp\left[\frac{i}{2}\int_0^{t'} dt\dot{X}^2(t)\right] \tag{28}$$

and the value of the action on classical trajectory

$$S_{cl} = t'(V^2/2 - sV) = x'^2/2t' - sx'. \tag{29}$$

Performing the integration, we find the Green's function far from the light cone

$$G_A(x,t) \simeq \sqrt{1/t'}[e^{ip_F\lambda x + i(x-st)^2/2t}\theta(x-st) + e^{-ip_F\lambda x + i(x+st)^2/2t}\theta(x+st)]. \tag{30}$$

The approximate equality sign refers to the functional determinant (in the massive directions) that was not calculated. We do not expect any radical effects from it, and postpone its calculation for a later research.

The main feature of this result is that the Green's function of quasiparticles of the CS model has a functional structure similar to that of free fermions [cf. Eqs. (13) and (15)]. In the limit of applicability of our analysis (i.e., far from the light cone), both correlation functions exhibit oscillations as a function of coordinate (x). However, the position of the light cone in the CS case is determined by the sound velocity and not by the bare Fermi velocity. In the case of strong interaction, this renormalization is quite large. We note that the value of the effective Fermi momentum for quasiparticle excitations is renormalized by the Calogero interaction as well. Though the correspondence between anions and solitons was known for quite a while, it is used here to derive an explicit result for the correlation function.

The last part of this work is devoted to the polarization operator of the CS model,

$$\Pi(q,t) = -i\langle T\rho(q,t)\rho(-q,0)\rangle. \tag{31}$$

The vanishing support of the polarization operator on the (ω,q) plane[23] is probably the most serious disadvantage of the TL model. As was pointed out in Ref. 1, one has to overcome this oversimplification in order to study various physical phenomena. Several attempts have been made to account for the finite support feature of one-dimensional interacting electrons. In the phenomenological approach,[24,25] the role of finite support is mimicked by the damping of collective excitations that result from the interaction between different modes.

As we show here, for the CS model, the finite width of the polarization operator's support may be attributed not to the *decay* of phonons but to the existence of many modes that fill that finite part of the frequency-momentum plane. As we know from the free fermion case, the polarization operator is determined by neutral particle hole excitations. The only neutral electron-hole excitations accounted for by the linear bosonization are acoustic phonons. In its nonlinear extension, the family of particle-hole excitation is much bigger and includes an infinite number of periodic density waves.[4,26] We claim that these excitations fill the continuum between the lower and upper boundaries of polarization operator support. Unlike decaying phonons, they correspond to exact many-body excited states of the system and have an infinite lifetime. To find a periodic density wave solution, one can use the saddle point Eq. (21) (but with zero at the right hand side), which for periodic functions may be rewritten as

$$\frac{v^2}{\lambda^2}\left(\frac{1}{\rho^2} - 1\right) + \pi^2\rho^2 + 2\pi\rho_x^H - \frac{1}{2}\frac{\rho_{xx}}{\rho} + \frac{1}{4}\frac{\rho_x^2}{\rho^2} = C, \tag{32}$$

where C is an arbitrary constant. Solving this equation,[4] one finds one-periodic solution

$$\rho_{B,l}(x) = 1 - \frac{1}{l} + \frac{1}{l}\frac{\sinh(2B/l)}{\cosh(2B/l) - \cos(2\pi x/l)}. \tag{33}$$

Here, l stands for a wavelength of the density wave, and B is the width of a single peak. Despite the complicated appearance, the periodic density wave is nothing but a periodic lattice of solitons. This observation is especially clear in a momentum representation

$$\rho_{B,l}(q) = e^{-B|q|/\pi}\sum_n \delta(ql - 2\pi n). \tag{34}$$

The velocity of the wave is expressed through the width of individual peaks and the distance between adjacent peaks as

$$v = s(1 - 1/l)\sqrt{1 + (4/l)(1 - 1/l)n_B(2B/l)}, \tag{35}$$

where $n_B(x)=1/(1+e^x)$. In a long length limit, this relation may be further simplified

$$v \simeq s + (2\pi\lambda/l)[n_B(2B/l) - 1/2]. \tag{36}$$

For the case where the width of the peaks is greater than the distance between them ($B \gg l$), the function n_B vanishes. Using the relation between wavelength of the density wave

and its wave vector ($q=2\pi n/l$), we recover a dispersion of anharmonic acoustic phonons $v(q)=s-\lambda q/2$. These excitations determine the lower edge of polarization operator support. In standard bosonization, only those are taken into account.[23]

For the waves with period longer than $l \gg B$, there is only a single peak (soliton) in that length. One can assume that since other peaks are far apart, one can study its motion regardless of the others. It contributes to the energy $E=\rho_0 v^2/2$. By recalling[27] the soliton dispersion [$\epsilon(q)=sq+q^2/2$, $q>0$], we obtain an upper support of the polarization operator. The form of support agrees with known results for rational values of coupling constants λ,[9] and it is an analytic continuation to generic values.[11]

To summarize, the soliton solution of hydrodynamic equations corresponds to anionic excitation of the many-body problem. By evaluating the value of the action on this solution, we calculated the single-anion Green's function. This function behaves similarly to the Green's function of the noninteracting electrons, but the scales (such as sound velocity and Fermi momentum) are renormalized by electron interaction.

Due to the infinite number of periodic density waves, a polarization operator acquires a finite support in the frequency-momentum plane, of which the lower edge of excitations is given by dispersive acoustic phonons (such excitation can be accounted for by linearizing an action, but accounting for phonon dispersion even for generic interaction). The upper edge is determined by a superposition of noninteracting solitons. As the interaction is varied from $1/r^2$, the solitonic excitations are destroyed and the upper part of the support is expected to smear.

Note added. Recently, such an investigation had been performed in Ref. 28, where it is indeed found that for generic interaction, the upper boundary of the support becomes smeared, while the lower boundary is unchanged.

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$$\Pi_{-}(q,\omega)=\frac{2\omega(q)S(q)}{[\omega-\omega(q)+i0\ \mathrm{sgn}\ \omega][\omega+\omega(q)+i0\ \mathrm{sgn}\ \omega]}.$$
It has a line $\omega=\omega(q)$ as a support on the ω,q plane. For free electrons, the polarization operator is given by
$$\Pi(\omega,q)=\frac{1}{\pi q}\ln\frac{(\omega+q+q^2/2)(\omega-q-q^2/2)}{(\omega-q+q^2/2)(\omega+q-q^2/2)}.$$
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